Q1a

1a

(a) Evaluate

$$\log_2 8^2 + 3\log_2 16 - 2\log_2 2^5.$$

$8^2 = (2^3)^2 = 2^6$, $16 = 2^4$

(b) Evaluate

$$3\ln 2 + 2\ln 5 - \tfrac{1}{2}\ln 10\,000,$$

giving your answer in the form $\ln p$.

[2]

[3]

a) rewrite all the functions as powers of 2

$\log_2 2^6 + 3\log_2 2^4 - 2\log_2 2^5$

rewrite powers as coefficients (or coefficients as powers!)

$6\log_2 2 + 3(4)\log_2 2 - 2(5)\log_2 2$

$(6 + 12 - 10)\log_2 2$

$8\log_2 2 = 8(1) =$ **8**

Q1b

1b

(a) Evaluate

$$\log_2 8^2 + 3\log_2 16 - 2\log_2 2^5.$$

(b) Evaluate

$$3\ln 2 + 2\ln 5 - \tfrac{1}{2}\ln 10\,000,$$

giving your answer in the form $\ln p$.

[2]

[3]

b) rewrite coefficients as powers

$\ln 2^3 + \ln 5^2 - \ln\sqrt{10000}$

$\sqrt{10^4} = 10^{\frac{4}{2}} = 10^2$

$(2\times 5)^2$

$\ln 2^3 + \ln 5^2 - \ln(2^2 5^2)$

Use log laws of addition & subtraction to combine

$\ln \frac{2^3 5^2}{2^2 5^2} =$ **ln 2**

Q2a

2a

(a) Solve the equation

$$4^{3x+2} = 16^{x+6}.$$

$16 = 4^2$

(b) Solve the equation

$$4^{2x+3} - 8 = 92$$

giving your answer to 3 significant figures.

[2]

a) rewrite 16 as a power of 4

$4^{3x+2} = (4^2)^{x+6}$

$= 4^{2x+12}$

[3]

equate powers

$3x + 2 = 2x + 12$

$x = 10$

Q2b

2b

(a) Solve the equation

$$4^{3x+2} = 16^{x+6}.$$

(b) Solve the equation

$$4^{2x+3} - 8 = 92$$

giving your answer to 3 significant figures.

[2]

b) $4^{2x+3} = 100$

$\log_4 4^{2x+3} = \log_4 100$

$2x + 3 = \log_4 100$

[3]

$x = \frac{(\log_4 100) - 3}{2}$

$= 0.161$ (3sf)

Q3a

3a

Solve the following equations, giving your answers in exact form.

(a) $4e^{3x-2} = 12$ [2]

(b) $3e^{2x} + 8 = 14e^{x}$ [3]

a) $e^{3x-2} = 3$

$\ln e^{3x-2} = \ln 3$

$3x - 2 = \ln 3$

$x = \frac{2 + \ln 3}{3}$

Q3b

3b

Solve the following equations, giving your answers in exact form.

(a) $4e^{3x-2} = 12$ [2]

(b) $3e^{2x} + 8 = 14e^{x}$ [3]

$(e^{x})^2$

b) $3(e^{x})^2 - 14(e^{x}) + 8 = 0$ ← quadratic in e^{x}!

let $e^{x} = y$

$3y^2 - 14y + 8 = 0$

$\times 24$, $+ -14$, $-12, -2$

$3y^2 - 2y - 12y + 8$

$(y-4)(3y-2) = 0$

$y = 4$ $\quad y = \frac{2}{3}$

$e^{x} = 4$ $\quad e^{x} = \frac{2}{3}$

$x = \ln 4, \ln \frac{2}{3}$

Q4a

4a

(a) Simplify

$2\ln 3^4 + \ln 3^3 - \ln 9,$

giving your answer in the form $a \ln b$, where a and b are integers to be found.

[2]

(b) Write

$$2\log_a x + 3\log_a (x+1) - \log_a 4(x+2)$$

as a single logarithm.

[2]

rewrite 9 as a power of 3

a) Using law ①

$\ln(3^4)^2 + \ln 3^3 - \ln 3^2$

$\ln 3^8 + \ln 3^3 - \ln 3^2$

Using laws ② & ③

$\ln \frac{3^8 3^3}{3^2} = \ln 3^9$

$= \boxed{9\ln 3}$

① $k\log_a x = \log_a x^k$

② $\log_a xy = \log_a x + \log_a y$

③ $\log_a \frac{x}{y} = \log_a x - \log_a y$

Q4b

4b

(a) Simplify

$2\ln 3^4 + \ln 3^3 - \ln 9,$

giving your answer in the form $a \ln b$, where a and b are integers to be found.

[2]

(b) Write

$2\log_a x + 3\log_a (x+1) - \log_a 4(x+2)$

as a single logarithm.

[2]

b) rewrite using ①

$\log_a x^2 + \log_a (x+1)^3 - \log_a 4(x+2)$

combine using ② & ③

$= \boxed{\log_a \left(\frac{x^2(x+1)^3}{4(x+2)}\right)}$

① $k\log_a x = \log_a x^k$

② $\log_a xy = \log_a x + \log_a y$

③ $\log_a \frac{x}{y} = \log_a x - \log_a y$

Q5

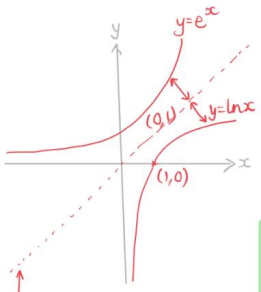
5

(i) On the same axes, sketch the graphs of $y = e^x$ and $y = \ln x$.
On each graph, label any points where the graph intersects the coordinate axes.
Write down the equations of any asymptotes for each graph.

(ii) Write down the line of reflection between the graphs $y = e^x$ and $y = \ln x$.

[5]

i)



x	-1	1	2	3
$\ln x$	X	0	0.7	1.1

math error +0.7 +0.4

as x increases, $\ln x$ increases at a decreasing rate

asymptote for $y=e^x$ is $y=0$

asymptote for $y=\ln x$ is $x=0$

ii) $x = y$ (line of reflection)

Q6

6

Solve the equation

$$5^{2x} - 8 \times 5^x + 12 = 0,$$

giving your answers in the form $\log_a b$.

[3]

$(5^x)^2 - 8(5^x) + 12 = 0$

quadratic in 5^x!

Let $y = 5^x$

$y^2 - 8y + 12 = 0$

$(y-6)(y-2) = 0$ (×12, +-8, -6, -2)

$y = 6 \quad y = 2$

$5^x = 6 \quad 5^x = 2$

$x = \log_5 6, \log_5 2$

Q7

7

Solve the equation

$$6 \times 3^{x-1} = 6^{2x},$$

giving your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers to be found.

[5]

$3^{x-1} = \frac{6^{2x}}{6} = 6^{2x-1}$

$= (2\times3)^{2x-1}$

$3^{x-1} = 2^{2x-1}\,3^{2x-1}$

$3^{(x-1)-(2x-1)} = 2^{2x-1}$

$3^{-x} = 2^{2x-1}$

$\ln 3^{-x} = \ln 2^{2x-1}$

$-x\ln 3 = (2x-1)\ln 2$

$x = \frac{1}{2+\frac{\ln 3}{\ln 2}} \times \frac{\ln 2}{\ln 2}$

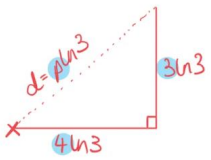
$x = \frac{\ln 2}{2\ln 2 + \ln 3} = \frac{\ln 2}{\ln 4 + \ln 3} = \frac{\ln 2}{\ln 12}$

Q8

8

A ship sets sail from a harbour.
After some time, the ship's position is $(4 \ln 3)$ km east of the harbour and $(3 \ln 3)$ km north of the harbour.
Find the direct distance between the ship and the harbour at this time giving your answer in the form $(p \ln 3)$ km.

[4]



Spot pythagorus' 3-4-5 triangle scaled by $\ln 3$ ∴ $p = 5$!

distance: $5\ln 3$

OR Alternatively use pythagorus' theorum to find the length of the hypothenuse...

$d = \sqrt{(4\ln 3)^2 + (3\ln 3)^2}$ $\quad d = 5\ln 3$

Q9

9

By writing 5 as $5 \ln e$, show that

$$5 \ln 2 + 5$$

can be written as $5 \ln 2e$.

[3]

$5\ln 2 + 5\ln e$ (1)

$= 5(\ln 2 + \ln e)$

Combine using log law

$= 5\ln 2e$

$\log_a xy = \log_a x + \log_a y$

Q10

10

Solve the equation

$$\log_3(x+4) = 4 + 2\log_3 x$$

giving your answers correct to 3 significant figures.

[3]

$\log_3(x+4) - 2\log_3 x = 4$

Apply law ①

$\log_3(x+4) - \log_3 x^2 = 4$

Apply law ②

$\log_3 \frac{(x+4)}{x^2} = 4$

$3^4 = \frac{x+4}{x^2}$

$81x^2 - x - 4 = 0$

$x = 0.228, -0.216$ (3sf)

reject negative

$x = 0.228$ (3sf)

① $k\log_a x = \log_a x^k$

② $\log_a \frac{x}{y} = \log_a x - \log_a y$

Q11

11

Solve the equation

$$2\log_x(x+2) = 3$$

giving your answer correct to 3 significant figures.

[3]

$k\log_a x = \log_a x^k$

$\log_x (x+2)^2 = 3$

$x^3 = (x+2)^2$

$x^3 = x^2 + 4x + 4$

$x^3 - x^2 - 4x - 4 = 0$

$x = 2.88$ (3sf)